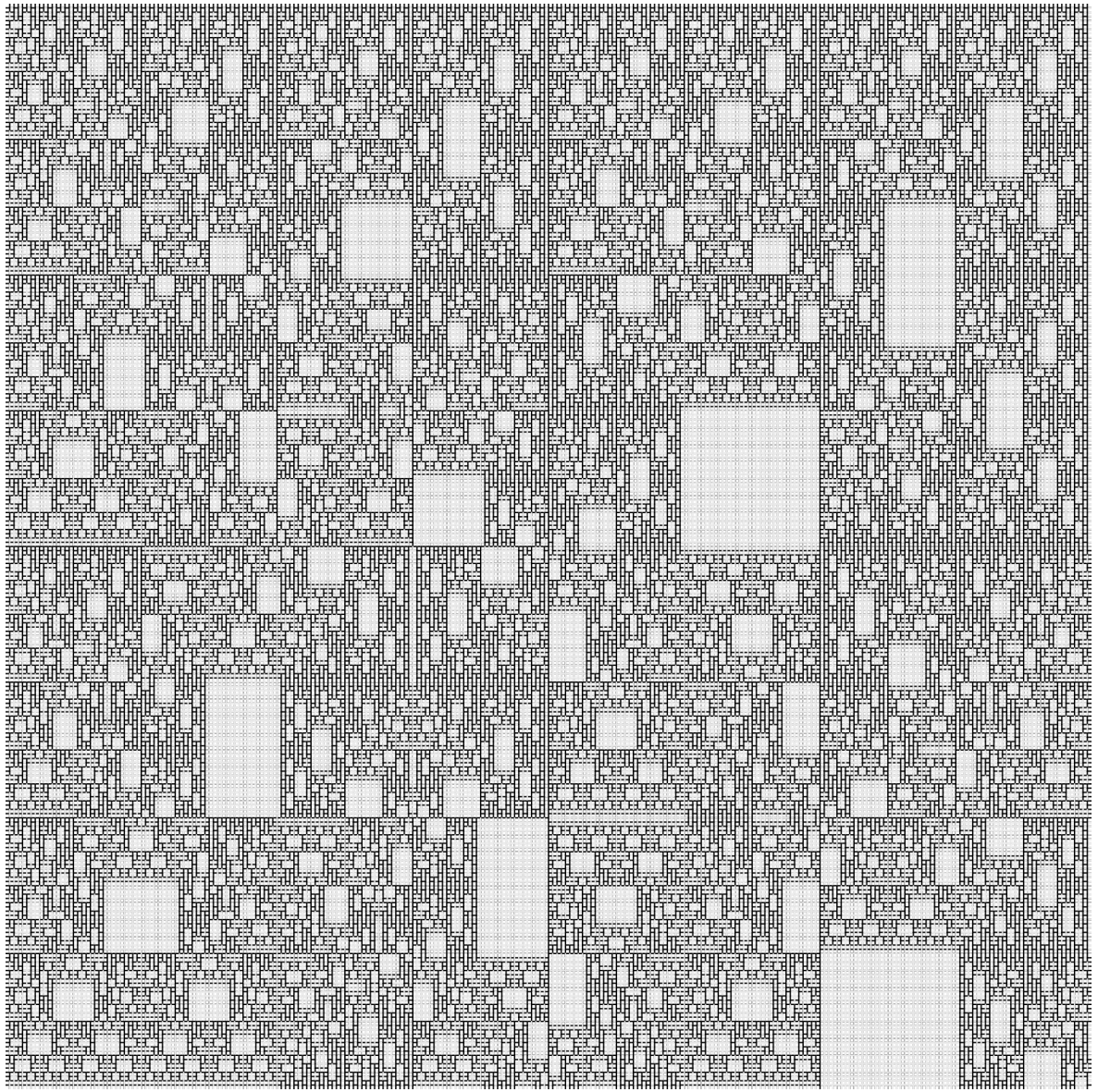
Binary Cyclic Codes with Polygons

T. Arthur Terlep

ROSE-HULMAN

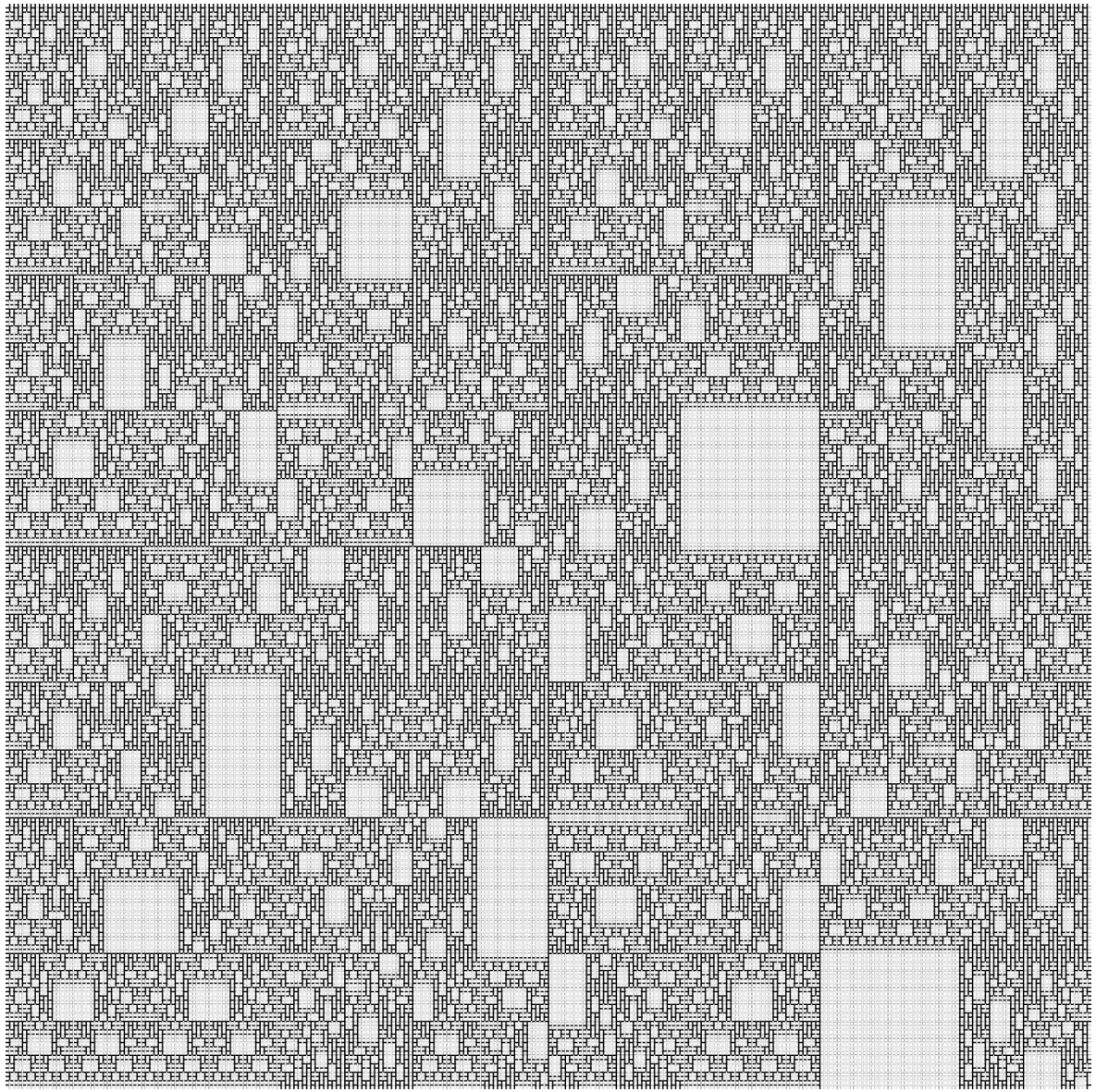
ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

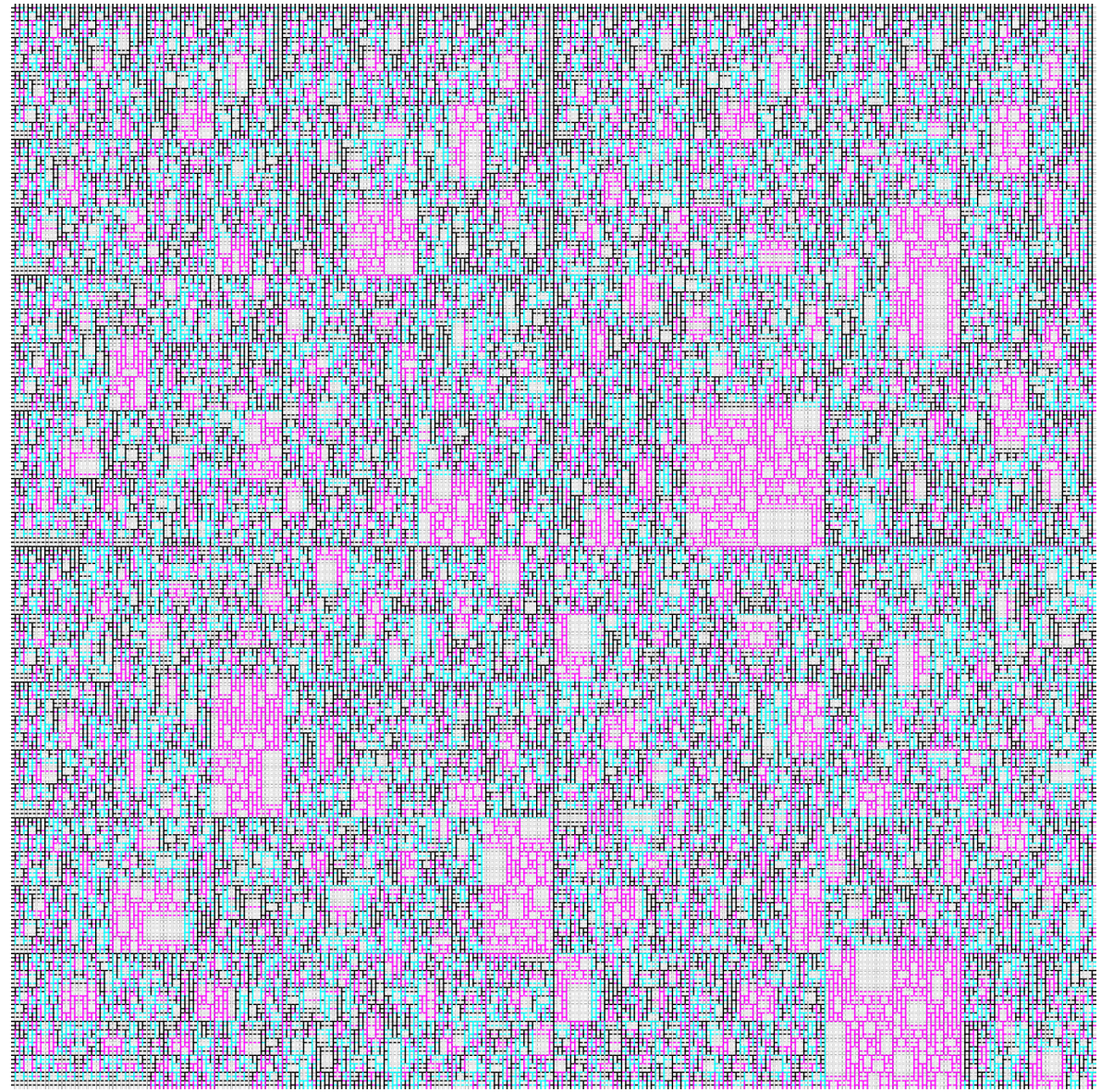
G4G16 Update
Jim Conant's fractal gasket updates



Lines are -1
Intersections are “multiply”

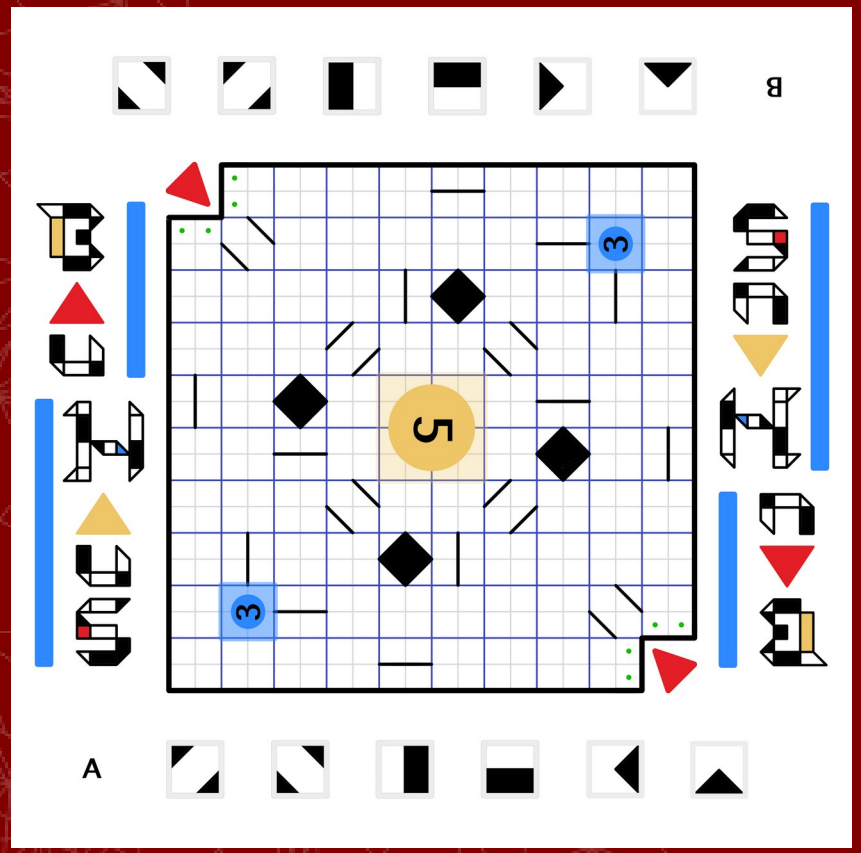
What about lines of i ?

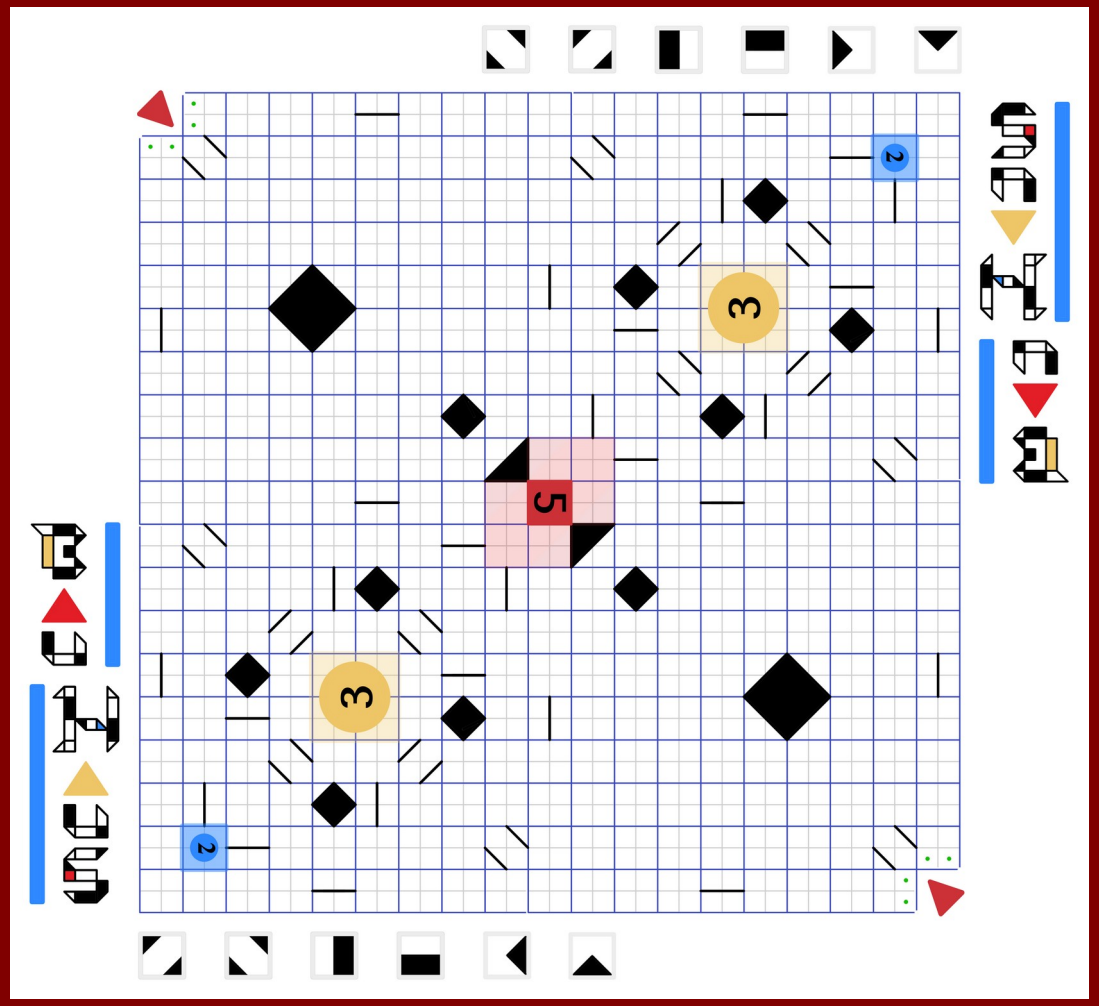




Gift exchange: Bauhaus game

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com





ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

Who's excited to learn about Galois Field Theory ?!

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

Who's excited to learn about Galois Field Theory?!

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

- What is an error control code?
- How can we visualize the most basic codes?
- I’m bored – what else do you have?

Error correction

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

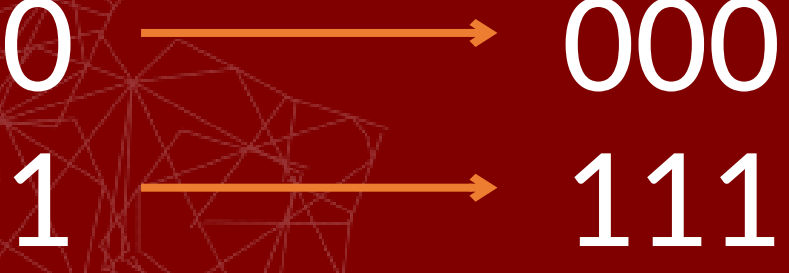
Send a message: 011
Oops! An error: 010

How can we fix it?

Dumb Error correction

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

Repetition “code”



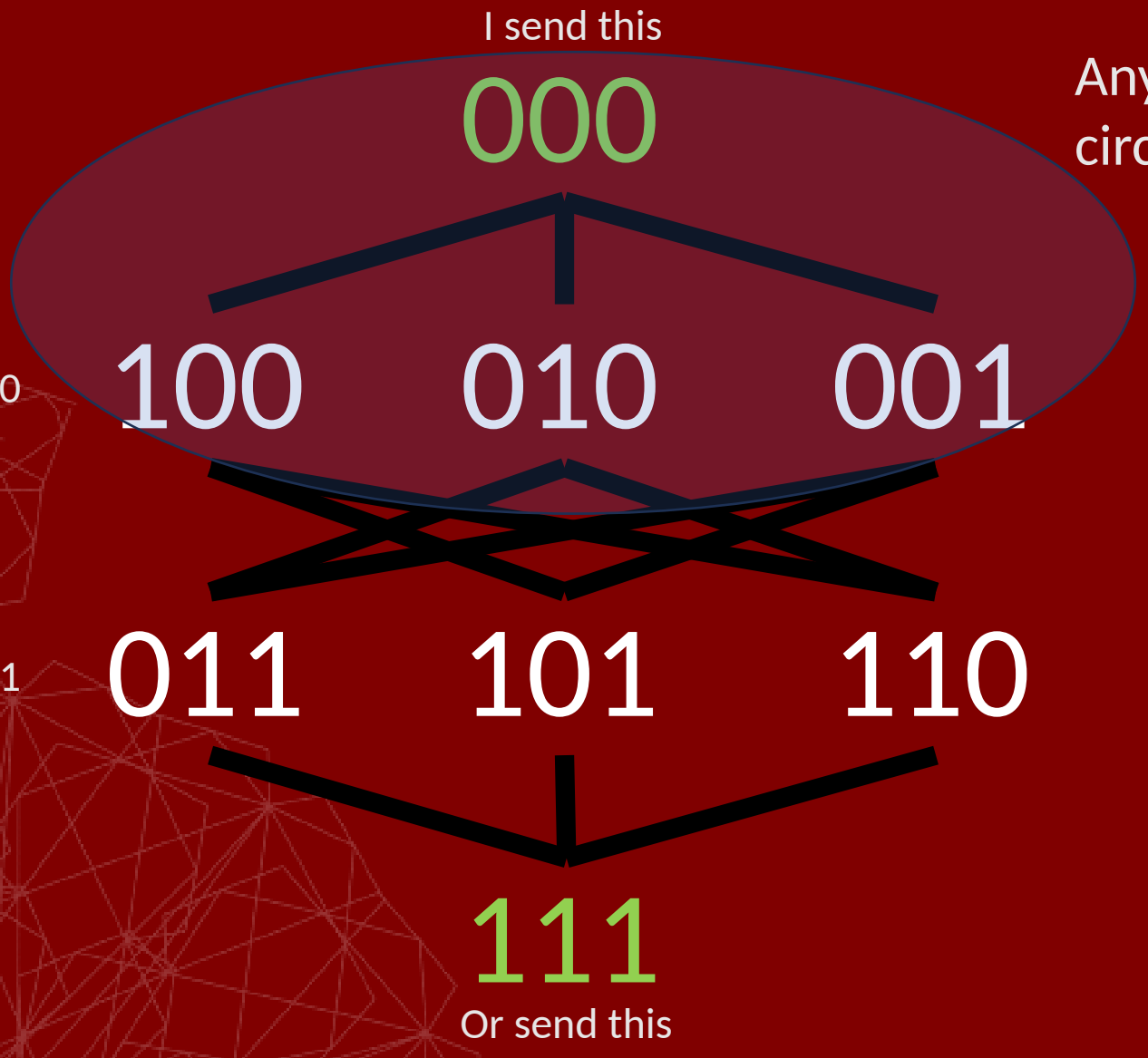
Now if we have an error, the two other bits can correct it! But it’s not efficient.

Dumb Error correction

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

Instead of “adding” more information bits, we can think of limiting the amount of information we *could* send in the first place.

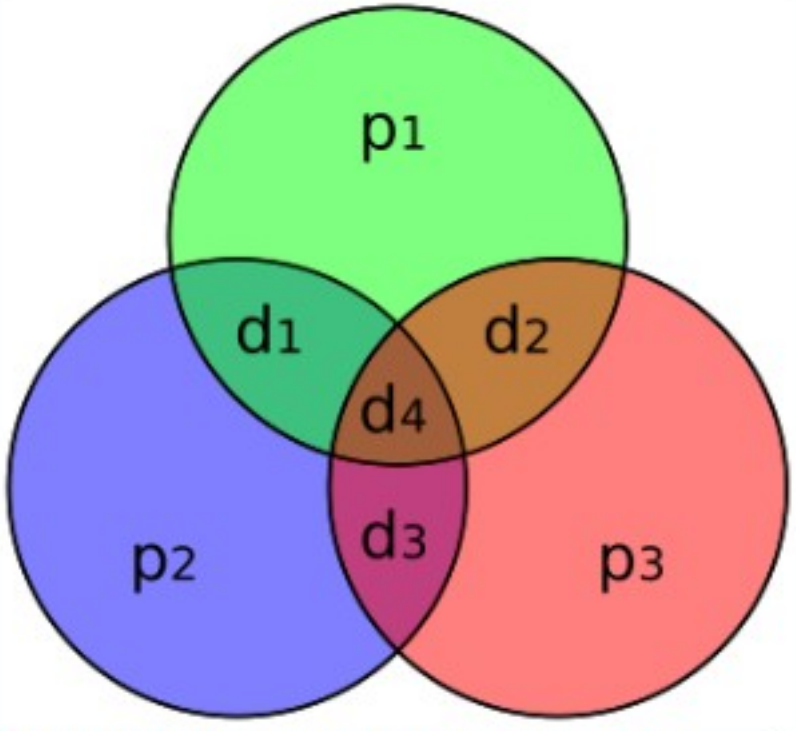
Dumb Error correction – Send a ‘0’ or ‘1’



Anything received in this circle is the message ‘0’

A known visualization

Hamming Code
7 bits total
4 information bits
3 error correction bits



Graphical depiction of the four data bits and three parity bits and which parity bits apply to which data bits

Binary Cyclic Codes

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

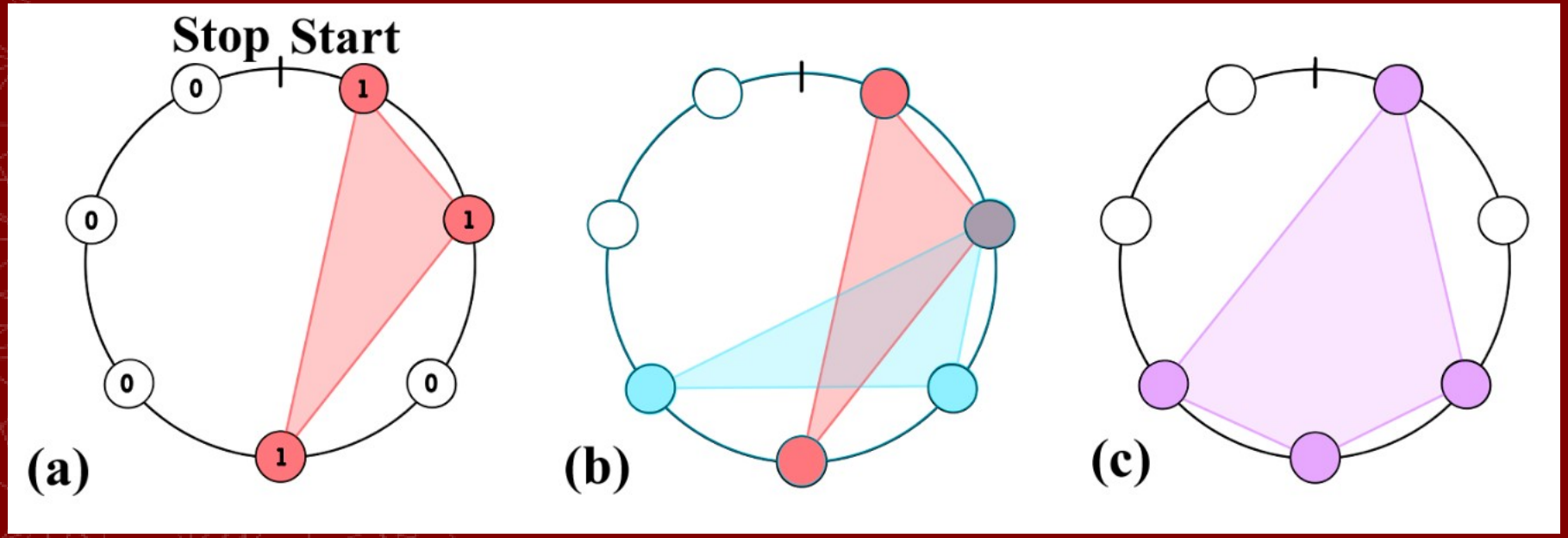
BCC follows 4 main rules for any code:

1. All zeros is a codeword
2. All ones is a codeword
3. The cyclic rotation of any codeword is also a codeword. 10011 ⟶ 11001 is also a codeword
4. Adding any two codewords is also a codeword.
   1+1 = 0, 0+0 = 0, 1+0 = 1, 0+1 = 0

A new way to visualize

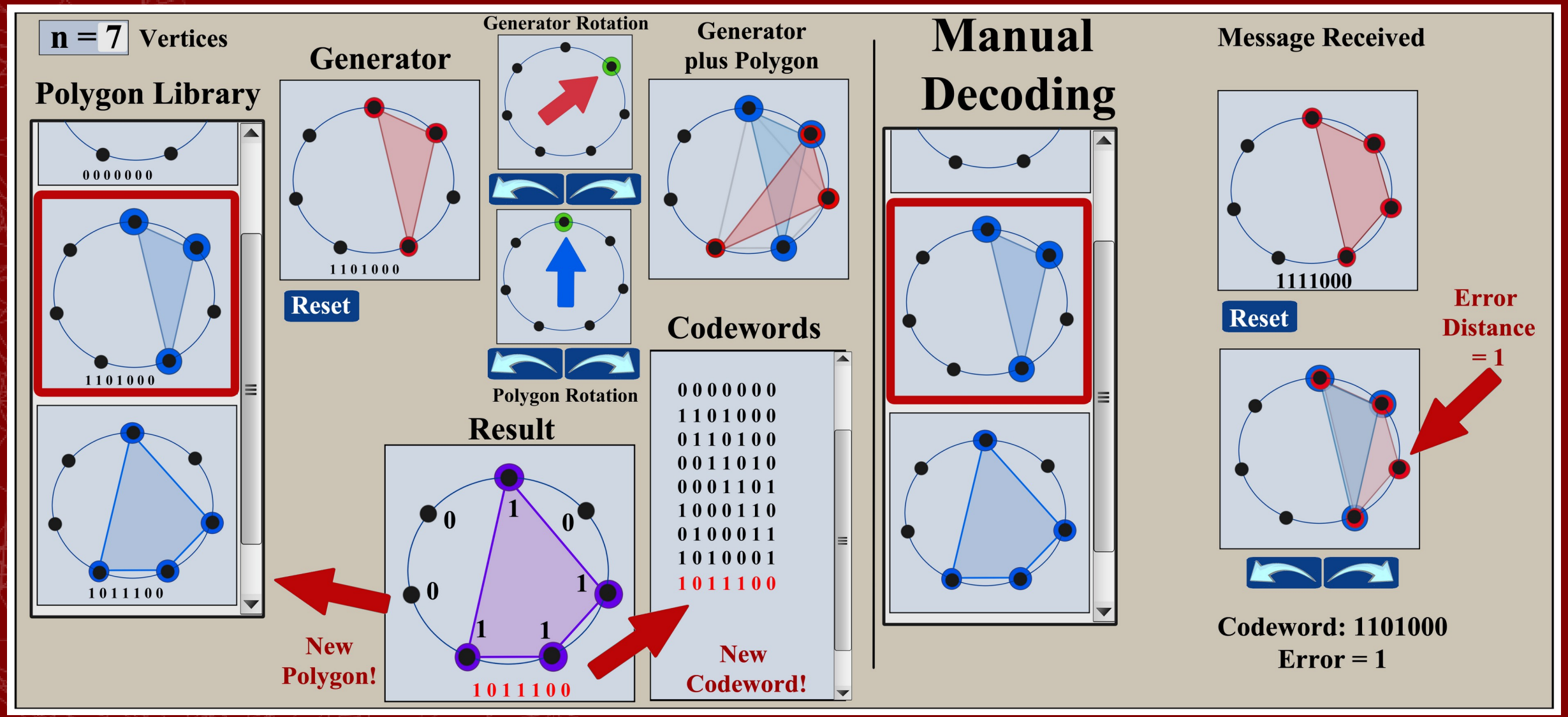
ROSE-HULMAN
INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

Suppose I have 7 bits to work with for messages.
Let 1101000 be a codeword. How can I find more?



New codeword! 1011100

WANTED: Looking for help making this visualizing educational applet!



And now for something… Completely different

ROSE-HULMAN INSTITUTE OF TECHNOLOGY
T. Arthur Terlep
taterlep@gmail.com

Let's talk about talking with graphs (Come see me!)

We begin with a toddler and some Tinker Toys . . .

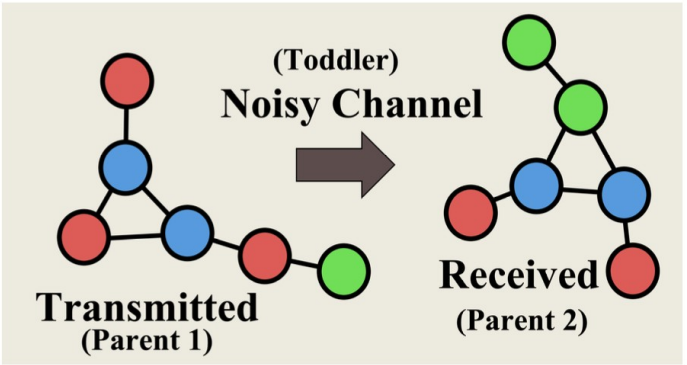


Figure 2.1. Tinker Toy and its toddler transmission as an analogy for the central problem of Graph-and-Symbol (GS) Communication. How can we effectively communicate using structures of symbols (colored objcts)?

A Mathematical Theory of Graph-and-Symbol Communication

T. Arthur Terlep, *Member, IEEE,* Mark R. Bell, *Fellow, IEEE,* and Thomas M. Langley

Abstract

Claude Shannon's foundational model of communication employed graphs to capture confusability among conventional time-indexed signals. However, relatively little attention has been given to communication scenarios where information is inherently structured, spatially or relationally, as graphs. In this work, we introduce a new mathematical framework called Graph-and-Symbol (GS) Communication in which symbols are assigned to the vertices of a graph. A message is defined not by a single labeled graph, but by an orbit of labelings under the action of the graph's automorphism group. This captures structural symmetry and semantic invariance. We introduce two coding schemes which are capable of correcting either a single symbol substitution or a single graph-edit error. The Graphadore-64 code is a nonlinear block code of length 7, comprised of 538 codewords drawn from 64 distinct graphs of order 7. Degree-parity codes are generalizable to any order $n \geq 5$ with 2088 codewords for $n = 7$.

Index Terms

Graph-and-Symbol (GS) channels, graph codes, error-correcting codes, graph automorphisms, combinatorial communication, memoryless channels, q-ary symmetric channel, structural information theory, orbit-stabilizer lemma.

I. Introduction

THE use of graphs in communication theory has a long-standing foundation, most notably in Shannon's original work on zero-error capacity, where graphs encode the confusability between input symbols. Recent developments have explored codes constructed from graph-theoretic relationships [1], but these treat the graph as a secondary structure imposed on an underlying linear message.

In contrast, we consider a framework in which the message itself is a graph with symbols assigned to its vertices. We call such a message a *Graph-and-Symbol (GS) word*. This combination of symbolic labeling and graph topology gives rise to a new class of communication models, which we refer to as *Graph-and-Symbol (GS) Communication*. Unlike prior models that emphasize sequential or time-indexed signals, GS-communication captures structured spatial, relational, or topological information transmitted as a unified object for a single use of the channel.



Presentation by T. Arthur Terlep

Thanks!

Questions? taterlep@gmail.com

Special thanks to:
Emma Terlep
Rose-Hulman Institute of Technology, ECE Department
Douglas Smith
Mark Bell
Les Shader

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY